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INFINITY MACHINES AND CREATION EX NIHILO

ABSTRACT. In this paper a simple model in particle dynamics of a well-known supertask is constructed (the supertask was introduced by Max Black some years ago). As a consequence, a new and simple result about creation ex nihilo of particles can be proved compatible with classical dynamics. This result cannot be avoided by imposing boundary conditions at spatial infinity, and therefore is really new in the literature. It follows that there is no reason why even a world of rigid spheres should be eternal, as has been erroneously assumed, especially since the time of Newton.

Let us take an infinite set of particles having the same mass which, at rest, occupy the points $x_i = \frac{1}{2^i}$ ($i \in \{1, 2, 3, \ldots\}$) in a unidimensional space. Let p_i be the particle that is at x_i. Perez Laraudogoitia (1996) has demonstrated that this system may self-excite spontaneously at a certain instant t^*, with the result that, after an infinite number of binary collisions, instant $t^* + \frac{1}{v}$ obtains, with the p_i particles at rest at the points $x_i = \frac{1}{2^{i-1}}$ ($i \geq 2$) and the particle p_1 at $x_1 = 1$ moving away from the origin 0 at velocity v. When this happens, I will say that the system has self-excited at t^* with velocity v and also that the excitation moves at velocity v. Naturally, t^* and v may assume any values whatever.

The preceding result presupposes that the particles interact solely through elastic collisions, and, although he did not state it explicitly, Perez Laraudogoitia made use of assumptions (A1) and (A2) below in his derivation, as well as of the temporal symmetry in the laws of mechanics:

(A1) As long as no collision occurs, a particle's velocity will remain constant.

(A2) If particle A, at velocity v, collides with particle B, at velocity $v = 0$, and both have the same mass, then after the collision A will move at velocity $v = 0$, and B at velocity v.

Temporal symmetry was used to justify the temporal inversion of a process which can be analysed solely in terms of (A1) and (A2), but it is easy to see that it is not needed. In fact, to justify the temporal inversion of such a process one need only show (A1) and (A2) to be invariants under temporal inversion, and it is obvious that they are. In effect, (A1) (a form of the first

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law of Newton) clearly does not refer to the sense of time, and would have an identical form in a time-reversed process (see Leggett (1987)). As for (A2), note that its temporal inversion, given in (A2*) – which amounts to changing the sign of the velocities involved, since velocity is the quotient between space and time – is logically equivalent to (A2) after a simple substitution operation of B for A and A for B, and one of replacement of v by $-v$:

(A2*) If particle B, at velocity $-v$, collides with particle A, at velocity $v = 0$, and both have the same mass, then after the collision B will move at velocity $v = 0$, and A at velocity $-v$.

Therefore, spontaneous self-excitation in the above sense requires only (A1) and (A2).

In the statement of (A2) there is an implicit assumption that particle B, say, emerges at velocity v after colliding with A at the very point of the collision, and not at some other point at an arbitrary distance from the former. This is natural, in presupposing that the trajectory of B in space-time (its so-called world line) is continuous during collision. In point of fact, it seems a commonsensical requirement that the world line of a particle should be continuous, if the notion that its identity is preserved over time is to make sense at all (see Quine 1976). Here we make this requirement explicit in the shape of an additional assumption, thus:

(A3) The world line of a particle is continuous. Namely, if particle p is at point x_0 at instant t_0, then for every neighbourhood V of x_0 there exists a neighbourhood U of t_0 such that the particle is at V at every instant $t \in U$ (see Jacobs 1992).

Consider now a denumerable infinite set of particles having the same mass, at rest at the points $x_{\pm n} = \pm\frac{n}{n+1}$ ($n \in \{0, 1, 2, 3, \ldots\}$) in a unidimensional space X. I will give the name $p_{\pm n}$ to the particle which is initially at rest at $x_{\pm n}$, so the particle at $x_{+0} = x_{-0} = 0$ can be called both p_{+0} and p_{-0}. The set of points x_{+n} is topologically equivalent to the set of points x_{-n} and both these sets are topologically equivalent to the set of points x_i mentioned above, which corresponds to the coordinates of a distribution of point particles for which spontaneous self-excitation is possible. Self-excitation can therefore occur for the set of particles p_{+n} as well as for the set p_{-n}. Obviously, excitation will move at negative velocity for the former and at positive velocity for the latter. Suppose now that the set of particles $p_{\pm n}$ self-excites spontaneously at instants $t_i = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^i}$ ($i \in \{1, 2, 3, \ldots\}$) at positive velocity $v_i = 2^{i+2}$ if i is odd and at negative velocity $v_i = -2^{i+2}$ if i is even. Such a very special form of self-excitation is, of course, unpredictable, but for our purposes suffice it to know that it is possible, that is, that it is compatible with (A1), (A2) and (A3). The time interval between self-excitations ith and $i + 1$th is obviously $\Delta t_i = \frac{1}{2^{i+1}}$ and in it self-excitation ith covers the open spatial interval (−1, +1) at a velocity of value 2^{i+2} in a time $t = \frac{2}{2^{i+2}} = \frac{1}{2^{i+1}}$. Consequently, self-excitation ith disappears precisely at the instant from which self-excitation $i+1$th originates. This disappearance is just the kind of process expounded by Perez Laraudogoitia (1996), whose temporal inversion gave rise to the self-excitation revealed there for the first time. It is obvious that every process of self-excitation-extinction has the effect of making each one of the particles $p_{\pm n}$ move a certain distance towards the right or towards the left. More specifically, if i is odd, self-excitation ith, which lasts for a time $\frac{1}{2^{i+1}}$, has the effect of placing

(a) particles p_{+n}, which at $t = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ were at rest at $x_{+n} = \frac{n}{n+1}$, at rest at $x_{+(n+1)} = \frac{n+1}{n+2}$ ($n \in \{0, 1, 2, 3, \ldots\}$)

(b) particles p_{-n}, which at $t = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ were at rest at $x_{-n} = -\frac{n}{n+1}$, at rest at $x_{-(n-1)} = -\frac{n-1}{n}$ ($n \in \{1, 2, 3, \ldots\}$)

If i is even, self-excitation ith, which lasts for a time $\frac{1}{2^{i+1}}$, has just the opposite effect, namely it places

(a) particles p_{+n}, which at $t = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ were at rest at $x_{+(n+1)} = \frac{n+1}{n+2}$, at rest at $x_{+n} = \frac{n}{n+1}$ ($n \in \{0, 1, 2, 3, \ldots\}$)

(b) particles p_{-n}, which at $t = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ were at rest at $x_{-(n-1)} = -\frac{n-1}{n}$, at rest at $x_{-n} = -\frac{n}{n+1}$ ($n \in \{1, 2, 3, \ldots\}$)

As a consequence of the above, all the particles oscillate increasingly quickly. Thus, particle p_{+n} is at $x_{+n} = \frac{n}{n+1}$ at instants $t_i = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ when i is odd, and at $x_{+(n+1)} = \frac{n+1}{n+2}$ at instants $t_i = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ when i is even ($n \in \{0, 1, 2, 3, \ldots\}$). As for particle p_{-n}, it is at $x_{-n} = -\frac{n}{n+1}$ at instants $t_i = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ when i is odd, and at $x_{-(n-1)} = -\frac{n-1}{n}$ at instants $t_i = \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^i}$ when i is even ($n \in \{1, 2, 3, \ldots\}$). Since the sum of the terms in an infinite succession $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots, \frac{1}{2^i}, \ldots$ is the unit, we can ask ourselves about the state of our system of particles at $t_0 = 1$. Let us suppose that particle p_{+n_0} ($n_0 \in \{0, 1, 2, 3, \ldots$) is at a certain point x_0 at $t_0 = 1$, and take as the neighbourhood V of x_0 an interval of length δ centred around x_0 (where δ is a number smaller than the difference $x_{+(n_0+1)} - x_{+n_0}$). Then it is obvious that there does not exist a neighbourhood U of $t_0 = 1$ such that p_{+n_0} is at V at every instant $t \in U$, because there exist instants of time t_i and t_{i+1} arbitrarily close to $t_0 = 1$ (it is enough to select an i which is sufficiently big) at which p_{+n_0} is at $x_{+n_0} = \frac{n_0}{n_0+1}$ and $x_{+(n_0+1)} = \frac{n_0+1}{n_0+2}$, that is, at points separated by a distance bigger than δ, and, as a consequence, at least one of them will lie outside V. Thus, the assumption that p_{+n_0} is at a certain point x_0 in unidimensional space at $t_0 = 1$ contradicts assumption (A3) that the world line of p_{+n_0} is continuous. Therefore, at $t_0 = 1$ p_{+n_0} cannot be at any point in space, but will have disappeared from it altogether. A similar line of reasoning obviously applies to particle p_{-n_0} $(n_0 \in \{1, 2, 3, \ldots\})$. Consequently, all the particles present at $t = 0$ will have disappeared at $t_0 = 1$ and space will be void of matter. It is easy to see that this result does not require that the particles be point particles. It could have been got by supposing that the particles are rigid spheres, though ones having progressively decreasing radii, the only remarkable modification that should be made being that in (A3) one should then refer to 'the world line of the geometric centre of a particle' instead of to 'the world line of a particle'. The further details can be easily worked out by the reader.

The result we have just got has philosophical relevance for two reasons at least. Firstly, because it throws light on a now classic problem introduced by Black (1950–51) involving what he termed 'infinity machines'. Black's paper is concerned with the movement of a marble (under the operation of two machines, Beta and Gamma), which is essentially identical to that of any of our particles $p_{\pm n}$ between $t = \frac{1}{2}$ and $t_0 = 1$, so what we have set forth in this paper so far is but a precise and detailed model of an infinity machine. Black reaches the conclusion that the increasingly rapid oscillating motion of his marble that he observes is incompatible with the continuous nature of movement. In his own words: "to say that motion is continuous is to deny that any real motion can be represented by a curve of this character" (Black 1950–51, 99). "For any material thing, whether machine or person, that set out to do an infinite number of acts would be committed to performing a motion that was discontinuous and therefore impossible" (Black 1950–51, 101). Black in fact goes further than this, as he adds that his infinity machines are logically impossible. That this latter statement is fallacious was revealed by Grünbaum (1969), who nevertheless accepts that the motion of the marble is incompatible with the continuity requirement when he speaks of "a kinematically impermissible discontinuity in the time variation of the position" (Grünbaum 1969, 243). Indeed, Grünbaum finishes his section on infinity machines with the following restatement: "the kinematically impermissible position pattern required by $\aleph_0$ transfers across a fixed distance in a finite time" (Grünbaum 1969, 244). What our model of infinity machine shows is that this conclusion of Grünbaum's is also mistaken. In effect, each particle performs a denumerable infinite number of oscillations while at the same time having a continuous world line, and there is no incompatibility in that. Naturally, this has one consequence, that at $t_0 = 1$ all the $p_{\pm n}$ have disappeared from space (Black's marble would suffer the same fate), which seems to violate a fundamental principle of the conservation of mass. This is perhaps the reason that led Grünbaum to see an unavoidable discontinuity in the problem posed by Black. The actual principle that is violated is the following:

(Q_1) for all time t_1 and t_2, the total mass at t_1 = the total mass at t_2.

Earman (1986) has demonstrated that (Q_1) is not a good way of formulating the principle of the conservation of mass (within the framework of classical physics, to which we have limited ourselves throughout this paper), because it takes no account of escape solutions, that is, situations in which a particle escapes to spatial infinity in a finite amount of time, thus disappearing from space. Earman replaces (Q_1) with the following formulation:

(Q_2) particle world lines do not have beginning or end points and mass is constant along a world line.

And it is obvious that our mechanical model of an infinity machine is consistent with (Q_2). In particular, particle world lines are topologically open: no point is the beginning or end point of a world line.

Using the temporal symmetry of the laws of mechanics, Earman offers an interesting demonstration of how escape solutions become situations in which a particle coming from spatial infinity can 'penetrate' into space in a spontaneous and unpredictable manner. He also points out that such a strange form of behaviour allowed by the laws of mechanics can be avoided laying down boundary conditions at infinity. To the same effect, Van Fraassen (1989) uses a picturesque example of a UFO and concludes that the universe is by definition isolated but not by definition conservative, that merely local theories about how individuals develop and interact cannot entail conservation (he is thinking of something similar to (Q_1)) for the universe as a whole. It is precisely in connection with this point that the second reason why the result obtained in this paper is philosophically relevant is to be understood. Since (A1), (A2) and (A3) are invariant under temporal inversion, the temporal inversion of the (unpredictable) process that leads to the disappearance of all the $p_{\pm n}$ at $t_0 = 1$ is also compatible with (A1), (A2) and (A3), and, therefore, possible and unpredictable. The fact that this process is possible can however no longer be blocked by means of any sort of conditions at spatial infinity. We would be just left with the following: a space void of matter in which, from a certain instant t^* there appear spontaneously a denumerable infinite set of particles of the same mass which move through mutual elastic collisions until they remain at relative rest at $t^* + \frac{1}{2}$ at points $x_{\pm n} = \pm\frac{n}{n+1}$, all of this obtaining without any influence from the coordinate regions $X > 1$ or $X < -1$. Therefore, even arbitrarily small regions of the universe can be isolated and yet not be conservative. Under these circumstances it would seem that one could refer properly to 'creation ex nihilo' of matter, thus refuting (at least one of the senses of) the well-known aphorism 'ex nihilo, nihil', which sums up Lucretius' philosophy. It is surprising that this can be achieved with such a meagre basis as offered by (A1), (A2) and (A3). There is no single instant, however, when creation can take place, since the topologically open nature of the world line of any particle implies that there is no first instant of its existence. Of course, we are stripping the term 'creation' of its metaphysical and/or theological connotations here, and it is evident that this is the only legitimate way to use it in our model of colliding particles. In current physical cosmology the technical details look very different, but the philosophical issue is essentially the same. This agrees with Grünbaum's (1989, 1991) recent emphasis on the logical hiatus opening between the problem of the origin of matter-energy or of the universe and the pseudo-problem of its creation by an external cause. There is no reason why even a world of rigid spheres should be eternal, as has been erroneously assumed, especially since the time of Newton.

Obviously, the theory that we have looked at here is of no cosmogonic import, but it does have conceptual significance. It helps to reveal the interesting connections between classical supertasks, which, like the one devised by Black, have always been viewed as no more than mere intellectual curiosities, and theoretically important results, which are beginning to be subjected to philosophical reflection.

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