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A response to a Platonistic and to a set-theoretic objection to the Kalam cosmological argument

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Abstract: The first premise of the Kalam cosmological argument has come under fire in the last few years. The premise states that the universe had a beginning, and one of two prominent arguments for it turns on the claim that an actual infinite collection of entities cannot exist. After stating the Kalam cosmological argument and the two approaches to defending its first premise, I respond to two objections against the notion that an actual infinite collection is impossible: a Platonistic objection from abstract objects and a set-theoretic objection from an ambiguity in the definition of '=' and '<' as applied to sets. The thought-experiment involving Hilbert's Hotel is central to the dialectic, and the discussion clarifies its use in supporting the Kalam cosmological argument.

In recent decades, there has been a veritable revival of activity in the philosophy of religion, and central to this revival has been renewed interest in theistic arguments, especially the cosmological argument.[1] There are three basic forms of the cosmological argument. First, there is the Thomist argument, which asserts the current existence of finite, contingent beings and proceeds, by way of rejection of an infinite regress of concurrent causes, to a *de re* metaphysically necessary being as the ground for the current existence of those contingent beings. Central to the argument is the distinction between essence and existence and the nature of the infinite regress – involving an essentially ordered series of causes – employed by its advocates. Second, there is the Leibnizian argument which begins with the question 'Why is there something rather than nothing?' and proceeds, by way of the principle of sufficient reason, to the truth of a *de dicto* logically necessary proposition 'God exists'. Central to the argument is the plausibility and range of application of the principle of sufficient reason.

The third argument is the Kalam cosmological argument. It is safe to say that this form of the argument has been more prominent in recent years than the other two, largely due to the writings of William Lane Craig.[2] Along with its rise in prominence has come a wave of criticisms of the argument. These criticisms have been stated in a number of places. For example, in a recent article by Wes Morriston, he casts a critical eye on just one part of Craig's case for the finitude of the past – viz. his philosophical argument against the possibility of actually infinite sets in the 'real world'.[3] In what follows, I shall state the Kalam argument, clarify two important objections – one Platonistic and one set-theoretical – and provide responses to them. As a Platonist who accepts the existence of an actual infinite number of abstract entities, I am initially sympathetic to these objections, but I believe that there are responses to them that preserve an important form of the Kalam cosmological argument, or so I shall argue.

A precis of the Kalam cosmological argument

The Kalam cosmological argument involves a defence of these three propositions:

(1) The universe had a beginning.
(2) The beginning of the universe was caused.
(3) The cause of the beginning of the universe was personal.

Elsewhere, I have defended (3) and, to a lesser extent, (2) but, in any case, my interest lies in (1).[4] So it shall be the proposition of concern in what follows. Two different philosophical arguments are typically offered on its behalf.

Argument A:
(A1) An actual infinite number of things cannot exist.
(A2) A beginningless temporal series of events is an actual infinite number of things.
(A3) Therefore, a beginningless temporal series of events cannot exist.
(A4) Either the present moment was preceded by a beginningless temporal series of prior events or there was a first event.
(A5) Therefore, there was a first event.

Argument B:
(B1) It is impossible to traverse an actual infinite by successive addition.
(B2) The temporal series of past events has been formed by successive addition.
(B3) Therefore, the temporal series of past events cannot be actually infinite.
(B4) Either the temporal series of past events is actually infinite or finite.
(B5) Therefore, the temporal series of past events is finite.
(B6) If the temporal series of past events is finite, there was a first event.
(B7) Therefore, there was a first event.

The notion of an actual infinite may be clarified according to these two widely used characterizations:

(A11) A set S is actually infinite if it is denumerable, that is, if it can be put into one-to-one correspondence with the set of natural numbers.

(A12) A set S is actually infinite if it can be put into one-to-one correspondence with a proper subset of itself.

Finally, there is one-to-one correspondence between two sets, S and T, just in case S and T have the same cardinality.

Notice that (A11) and (A12) merely offer sufficient conditions for a set being an actual infinite. This is all that is required for Arguments A and B, and this will be important for evaluating the set-theoretic objection below. For now, I merely note that the point of (A12) is not in the identity of members included in the proper subset; rather, it is in the cardinality of that subset. This may be seen by adjusting (A12) as follows:

(A12′) A set S is actually infinite if it can be put into one-to-one correspondence with a set T that can be put into one-to-one correspondence with a proper subset of S.

Arguments A and B are epistemically separate in at least this sense: one may justifiably accept B without so accepting A. It is not clear that the converse is true. Of course, one may accept A without accepting B, but it is not clear that one may justifiably do so. In this article, I am concerned with two objections raised against (A1), so argument B is not relevant to the present dialectic. Before we can understand those objections, it is important to get before us the main argument for (A1) they target. The argument turns on the claim that if it is granted that an actual infinite set of things exists, then there are unacceptable implications that follow and, thus, we ought to reject the existence of an actual infinite set of things. Here is Craig's statement of the argument in terms of a well-known illustration, Hilbert's Hotel:[5]

> Let's imagine a hotel with a finite number of rooms. Suppose, furthermore, that all the rooms are full. When a new guest arrives asking for a room, the proprietor apologizes, 'Sorry, all the rooms are full'. But now let us imagine a hotel with an infinite number of rooms and suppose once more that *all the rooms are full*. There is not a single vacant room throughout the entire infinite hotel. Now suppose a new guest shows up, asking for a room. 'But of course!' says the proprietor, and he immediately shifts the person in room #1 into room #2, the person in room #2 into room #3, the person in room #3 into room #4, and so on, to infinity. As a result of these room changes, room #1 now becomes vacant and the new guest gratefully checks in. But remember, before he arrived, all the rooms were full! Equally curious, according to the mathematicians, there are now no more persons in the hotel than there were before: the number is just infinite. But how can this be? The proprietor just added the new guest's name to the register and gave him his keys – how can there not be one more person in the hotel than before? But the

situation becomes even stranger. For suppose an infinity of new guests show up at the desk, asking for a room. 'Of course, of course!' says the proprietor, and he proceeds to shift the person in room #1 into room #2, the person in room #2 into room #4, the person in room #3 into room #6, and so on out to infinity, always putting each former occupant into the room number twice his own. Because any natural number multiplied by two always equals an even number, all the guests wind up in even-numbered rooms. As a result, all the odd numbered rooms become vacant, and the infinity of new guests is easily accommodated. And yet, before they came, all the rooms were full! And again, strangely enough, the number of guests in the hotel is the same after the infinity of new guests check in as before, even though there were as many new guests as old guests.

A Platonistic objection to the Kalam cosmological argument

The Platonistic argument has been repeatedly levelled against (A1) and it may be summarized in this way:

(P1) If Platonism about abstract objects is true, then an actual infinite number of things exists.
(P2) Platonism about abstract objects is true.
(A1) An actual infinite number of things cannot exist.
(P3) Therefore, (A1) is false.

A defender of (A1) could respond by rejecting (P2). Since I have defended Platonism elsewhere, that option is not available to me and, in any case, I am interested in defending (A1) for those who are either epistemically counterbalanced with respect to (P2) or no more than mildly inclined against it, especially for those who would accept Platonism if they did not think it would require abandoning (A1). That leaves (P1). Before I comment on it, I should say more about Platonism.

As I am using the notion, Platonism entails a commitment to the existence of abstract objects, where 'abstract object' is straightforwardly ontological.[6] In this sense, an entity is abstract just in case (a) it is not a person, and (b) it exists outside space and time in that it has no spatial or temporal location or duration. So understood, an abstract object is an immutable, necessary being.[7]

The idea that Platonism entails an actual infinite number of entities is sometimes supported by various regress arguments involving universals, though the problem of an infinity of abstract objects arises from a commitment to abstract objects (for example, sets) besides universals. But since universals are relevant to arguing for (P1), it is important to get clear on what a universal is. The core aspect of the notion of being a universal is this:

(U) Some entity E is a universal just in case E as such is multiply exemplifiable.

Four comments need to be made about (U). First, it takes exemplification to be primitive and defined ostensively. Second, it characterizes a universal as an entity that is exemplifiable *qua universal*. This qualification is needed because some realists regarding universals are worried about properties, such as being circular and square, which are not exemplifiable. It is possible to reject such properties (though one could still embrace a concept of being circular and square which is itself neither circular nor square), but if a realist accepts them, it is open to him/her to hold that such a property (e.g. being circular and square) *qua* universal is exemplifiable but that this feature of the property is overridden by a different metaphysical principle, e.g. necessarily, two determinates under the same determinable cannot be co-exemplified by the same object at the same time. Thus, being circular and square is exemplifiable *qua* being a universal but not *qua* being two determinates under the same determinable.

Third, the essence of a universal is that it is exemplifiable, not that it is abstract. Being abstract is not sufficient for being a universal (sets are not universals), and, it is epistemically possible that being abstract is not necessary for being a universal. Thus, D. M. Armstrong's theory of universals as multiply exemplifiable entities is spelled out in terms of a universal being a multiply located entity fully present at each space-time place occupied by the particular that exemplifies it.[8] Having said that, I believe Armstrong is wrong and I have criticized his position elsewhere and argued that universals are, indeed, abstract objects.[9] Moreover, if Armstrong is correct, then in combination with other arguments (e.g. Aristotelian requirements that universals must be exemplified to exist) one could embrace universals without having to commit oneself to an actual infinite number of them, so I will set his view aside for that reason and accept the claim that universals are abstract objects.

Finally, 'exemplifiable' is used in a somewhat idiosyncratic way to mean 'possibly exemplified and need not be exemplified'. The second conjunct is not entailed by the first conjunct. If something is actually exemplified, then it is possibly exemplified and an Aristotelian could accept the first conjunct. But since Platonists embrace the existence of unexemplified universals, the locution above is the correct analysis of 'exemplifiable' as it is used in (U). I could have used the longer locution in (U) but, for brevity's sake, I have chosen the more idiosyncratic notion.

Why think that Platonic realism requires a commitment to an actual infinite number of universals? Among the arguments employed to establish the Platonic commitment to an infinite number of abstract entities, two stand out in recent discussions. The first derives from what is called a maximalist conception of properties, according to which there are as many properties as there possibly can be.[10] If a property could exist it does exist. One argument for this conception of properties is the claim that since properties are necessary beings, they exist throughout possible worlds, so if a property is possible, it must be necessary and, thus, actual.

A second argument derives from mathematics:[11]

(M1) Propositions expressed by sentences in number theory are often true or false.
(M2) Such propositions necessarily have the truth values that they do.
(M3) The numerals in a sentence like '$7+5=12$' are genuine, denoting singular terms that refer to abstract objects as the truth-makers for propositions expressed by sentences in number theory.
(M4) There is an actual infinite number of mathematical truths and abstract objects which are their associated truth-makers.

The maximalist conception of properties is controversial, and not all realists accept it. But the argument from mathematical truth is a powerful one. I think that it, or something like it, lies behind most employments of the Platonistic argument against (A1). Thus, in his critique of (A1), Wes Morriston claims that 'since the number of mathematical truths (to say nothing of all the other eternal truths concerning properties and propositions and the like) is clearly infinite, it follows – does it not? – that an *actual* infinity is present in God's knowledge'.[12] Notice that (M3) does not require that the referents of numerals be universals; all that is required is that they be abstract objects.

A defender of (A1) could reject (M4), but I will not pursue that move because I don't think it is plausible and I am interested in defending (A1) to those who accept (M4). What, then, can one say to rebut the Platonistic argument against (A1)? In my view, the problem does not lie with (P1) or (P2); rather, the difficulty seems to involve the way (A1) is stated in the argument. To see this, we need to return to Craig's argument on behalf of (A1).

Craig's support of (A1) appeals to two different situations that arise with respect to Hilbert's Hotel that are intuitively problematic. First (1), granting a full hotel with an actual infinite number of rooms, if a new guest wants to check in and the proprietor responds by shifting all the current guests to a different room, a new room is opened up for the new guest. As Craig points out – correctly, in my view – this results in an absurdity: the envisioned scenario is impossible because all the rooms were full. There were no empty rooms available for the shift to take place and, thus, no possibility of opening up a new room. And given that the hotel has an infinite number of full rooms, each of which occupies some finite spatial extension *s*, the hotel extends infinitely far into distant space. In this case, there is no space available for adding new rooms to the hotel or into which guests can be shifted.

Second (2), if the scenario just mentioned were to take place, it would generate the absurd conclusion that there are no more guests in the hotel than before. But this cannot be, says Craig – again, correctly, in my view – because a new guest has just been given keys to check into the hotel.

Craig's concludes from these (and related) puzzles by noting that a fundamental axiom of trans-finite mathematics – the denial that a whole is greater than any of its proper parts – generates 'all sorts of absurdities, like Hilbert's Hotel, when one tries to translate that theory to reality'.[13]

The problematic nature of situation (1) seems to follow from two things. First, the members of the infinite set are finite, located, moveable entities. This opens up the possibility of adding, subtracting, or rearranging the members of the set.

Second, the members of the set are spatially extended. This second feature would generalize to sets whose members were either spatio-temporally or temporally extended, but since Hilbert's Hotel relevantly involves members that are spatially extended, I will limit my remarks here to spatial extension.[14] If an object *e* is spatially extended throughout and only throughout some location L, then both (i) if *e* has proper parts, they overlap with and only with sub-regions of L, and (ii) neither *e* nor any of its proper parts overlap with some other location P such that P is not identical to L or a sub-region of L. On the assumption that there are ultimate atomic simples of spatial extension, then given some spatial atom *l* and some object *e* co-extensive with and only with *l* and located at and only at *l*, then *e* is spatially extended in that *e* overlaps with and only with *l*. Appropriate adjustments could be made to this characterization to allow for spatial regions themselves to exist throughout or at an extended location. Since $\aleph_0$ times any finite number *n* is equal to actual infinity, if there is an infinite number of rooms each of which is finitely extended, then the hotel extends forever, infinitely far into the distance.

Regarding Hilbert's Hotel, the problem is that if we move the guests from one location to another, there just are no rooms available into which they can be moved. All of them are already filled. Moreover, there is no way to open up a new room by this procedure because there is no spatial region available into which they can be moved or new rooms can be added. The hotel extends infinitely far into the distance.

The problematic nature of situation (2) is more ambiguous, but it would seem to be one or both of the following: (a) if one adds (or subtracts) members to an actual infinite set, then one has not increased the number of members of that set, but this is false since we have before us the new member who was added; (b) it is just self-evident that a whole is greater than any of its proper parts, and the thought-experiment involves a whole (the set of guests after the new one checks in) that is alleged to be equal to one of its proper parts (the original set of guests).[15]

I think a Platonistic defender of (A1) can say that the two situations used to support it both turn on the fact that the members of the set are (i) finite, contingent entities that can be added to (rearranged within or subtracted from) a set, and (ii) spatially (or spatio-temporally or temporally) extended. This clearly seems to be the problem in situation (1). It is the fact that we are supposed to have an infinitely large hotel completely filled with guests, who move into new locations that are not available, and that then opens up a new room, that could not be made available, that generates the absurdity in this thought-experiment. Where are the guests supposed to go? How can such a shift take place? How could the hotel open up a room when there just are no rooms or space available? Arguably, these are the problematic issues for situation (1).

Similarly, the Platonist will argue that it is (a) not (b) *per se* that generates the intuitive implausibility in situation (2). I will say more about the issues involved in clarifying (a) later when I discuss the set-theoretic objection to (A1). For now, it will suffice to say that by appealing to (a) as the problem, the Platonist is saying that it is the fact that actual, live guests are being added to (or subtracted from) the hotel that makes the situation implausible. If I am right about this, then a Platonistic defender of Argument A for the Kalam cosmological argument will adjust (A1) to read as follows:

> (A1′) An actual infinite number of finite, contingent entities that (1) can be added to or subtracted from a set and (2) are spatially (or spatio-temporally or temporally) extended cannot exist.

(A1′) allows Argument A to go through in a manner consistent with a Platonistic view of abstract objects as non-spatio-temporal, immutable, necessary beings. So understood, an abstract object cannot be added to or subtracted from anything, so they are not proper candidates for members of sets included in thought-experiments employed against the existence of actual infinite collections. Further, abstract objects are neither spatially (or temporally) located or extended, so there is no need to find room for them next to each other or at some other location. And (A1′) allows one to accept Craig's claim that the denial that a whole is greater than any of its proper parts generates 'all sorts of absurdities, like Hilbert's Hotel, when one tries to translate that theory to reality'. As (A1′) makes clear, the problem surfaced by Hilbert's Hotel is not the attempt to apply the axiom of transfinite mathematics to reality *per se*, but rather, the attempt to apply it to the realm of concrete objects in which entities are finite, contingent relevantly extended entities.

But what about (b) itself? It does seem that the Platonist must deny (b) *per se*. The Platonist will say that, in light of thought-experiments such as Hilbert's Hotel, it becomes obvious that in the realm of concrete objects as characterized in (A1′), a whole is always greater than any of its proper parts, but that this is not the case in the realm of abstract objects. Perhaps the Platonist can argue that someone's misguided confidence in applying this axiom to the realm of abstract objects derives from an attempt to image or picture, say, a vast expanse of numbers ordered next to each other in space such that a disregard for certain numbers, e.g. the even numbers, leaves holes in the expanse analogous to taking the even numerals away.

Since my purpose is merely to undercut the use of Platonism to provide grounds for rejecting Argument A, this further insight about the source of confidence in applying the axiom to the abstract realm may not be required. However, before we turn to examine a different criticism of the Kalam cosmological argument, it is important to examine some interesting remarks by Wes Morriston relevant to the current discussion.[16]

Morriston notes, correctly in my view, that if set S is impossible because some absurdity follows from features *a*, *b* and *c* of set S, this does not imply that no set having feature *a* is possible. Rather, it follows that no set with all three features *a*, *b* and *c* is possible. By way of application, Morriston claims that from certain absurdities to which Craig has called our attention, it does not follow that infinite sets in general are impossible. Rather, the absurd implications follow from the way in which the actual infinite number of elements in the set interacts with other features of the thought-experiment. So far, so good. This is precisely what I have been arguing.

However, I part company with Morriston when it comes to his positive identification of these other features. He claims that they involve 'a collection of *coexistent* objects … whose physical relationship to one another can be *changed*. It is only when these features are *combined* with the property of having infinitely many elements that we get [absurd implications]'.[17] According to Morriston, if the infinitely many elements (books in an infinite library, rooms/guests in an infinite hotel) and spaces (shelves/rooms) did not exist at the same time, there could be no thought of rearranging them. So, even if we grant that certain infinite collections of simultaneous coexistent objects is impossible, the same cannot be said for an infinite series of past events: '[E]vents that have happened are fixed in their temporal locations. They cannot be changed or rearranged in such a way as to open or close temporal locations.'[18] Thus, for Morriston, the coexistence of 'physical' entities is a necessary condition for those entities being changed relative to each other or to some other background grid.

For two reasons, I think Morriston is wrong about his claim that Craig-type absurdities cannot be applied to an infinite set of past events: he is wrong that the coexistence of the elements of an infinite set is a necessary condition for generating the absurdities and he is wrong that past events could not have been differently arranged or changed.

I have already defended my view in connection to what I take to be Morriston's first mistake. It is not the coexistence of the elements that creates the problems. It is the fact that the elements of the infinite set are finite, contingent, moveable entities that can be added or subtracted and that exemplify the relevant sort of extension that generates the absurdities. When this happens in the hotel or library thought-experiments, it creates allegedly fillable gaps that cannot, in fact, be filled, or it requires moving/adding things for which there is no room, or it implies that we cannot add a new book or guest when we have one right before us.

My case against Morriston's first claim may be further supported by examining his second claim, viz. that since past events are fixed in their temporal locations, they could not have been differently arranged. It may well be the case that, on an A-series view of time, given the present moment, one cannot go back and change the past. If this is what Morriston is claiming, his assertion is true but irrelevant to the issues before us. This is because it does not follow from this concession alone (setting aside Craig-type absurdities) that logically prior to creation, God could not have created a world with the same series of moments as occurred in the actual world prior to some time t_0 but with, say, five additional years added prior to what was the first event in that series of moments. Nor does it follow that God could not have created a possible world in which certain moments of time in our world were relocated temporally so as to occupy different temporal locations. All that is needed for such a thing to be possible is that temporal moments do not stand in internal relations to other moments.

Thus, from the fact that an already actualized past cannot be changed relative to some present time t, it does not follow that the temporal location of moments prior to t could not have been different or that those moments could not have been rearranged. In the example of Hilbert's Hotel, given the actual existence of an infinite collection of identifiable guests, it is the mere possibility of adding new guests to, or rearranging guests within, the hotel that generates the puzzles. Similarly, given the actual history of the cosmos with a set of identifiable moments prior to some time t_0, it is the possibility that moments could have been added to or rearranged relative to that identifiable set that sustains the parallel with Hilbert's Hotel, even if we agree that given t_0, it is not at that time possible to go back and change the past prior to t_0.

If a theory of individuation is plausible in which temporal moments *qua* particulars are depicted as standing in external temporal relations to other temporal moments, that would be sufficient to rebut Morriston's claim about the fixed location of temporal events. And it would be sufficient to rebut his claim that the coexistence of two entities is a necessary condition for their rearrangement relative to each other or some background grid. Space considerations do not allow me to develop such an account of individuation in detail here. I have addressed problems of individuation elsewhere and can only gesture at a few points in the present setting.[19]

For the sake of argument, let us grant a property-exemplification theory of events (or a near cousin), according to which an event is a whole that contains among its constituents at least one property. On a certain version of an A-series view of time, each moment has the property of presentness. Granting that presentness is a universal exemplified by each present moment, it would follow that each moment could not be individuated by having the property of presentness. In addition to this property, each occurring present moment would need to contain an individuator as a constituent. I have argued elsewhere that bare particulars are the relevant individuators. Granting this solely for the purposes of illustration, each present moment is identical to an event whose constituents are presentness, exemplification, and a bare particular (or some other individuator).

If this or some relevantly similar view is correct, there is no reason why God could not conceive of a particular individual moment as existing at a different temporal 'location' in different possible worlds, and bring these different states of affairs about. Each moment of time is constituted by identity conditions that merely involve constituents within that moment (presentness, exemplification, an individuator). Temporal relations among moments are external. This does not imply that some event e_1 at time t_1, say an apple turning red at t_1, could have occurred at another time. Of course, the apple could have turned red at a different time, but since t_1 is an essential constituent of e_1, the latter could not have occurred at some other time.

On a B-series view of time, time is closely analogous to space. I see no incoherence in the idea that spatial locations, e.g. simple spatial points, could not stand in different spatial relations to other spatial locations. For example, the particular spatial location that currently overlaps one of my atomic simple parts as I write this could retain its identity if the universe were half the size is actually is. Similarly, I see no incoherence in the idea that a particular temporal moment on a B-series time line could retain its identity in possible worlds in which the time line were shorter or longer, or even in which that temporal point were located at a different place in the time line.

It seems, then, that there are reasons to doubt Morriston's claim that events are fixed in their temporal locations, if that assertion is charitably interpreted in a way relevant to Craig-type absurdities. If so, then the same kinds of absurdities that characterize infinite hotels or libraries can be generated in scenarios in which infinite sets of temporal moments have moments added to or subtracted from them. Thus, it is not the coexistence of the elements of infinite sets that generates the problems. It is both their finite, contingent, moveable nature and the fact that they exemplify the relevant sort of extension.

In any case, for a Platonist who is committed to an actual infinite number of abstract objects and who also accepts Argument A, it would seem that (A1′) is the way to go. It allows thought-experiments such as Hilbert's Hotel to retain their intuitive force without requiring an abandonment of Platonism, and it undercuts the use of the latter as justification for rejecting the first premise of Argument A and the thought experiments used to justify it.

A set-theoretic objection to the Kalam cosmological argument

A second criticism that has been raised against (A1) derives its force from different set-theoretic definitions of '=' and '<' as applied to sets.[20] Given two sets, A and B, suppose we want to know if $A=B$ or if $A<B$. The difficulty in answering this question comes from an ambiguity in defining '=' and '<'.

Definition 1 $A=B$ if every element of A is in B, and every element of B is in A. $A<B$ if every element of A is in B, and some element of B is not in A.

Thus,

$\{1,2,3\}=\{2,1,3\}$
$\{1,2,3\}\neq\{3,8,6\}$
$\{1,2\}<\{1,2,5\}$
$\{1,2\}\nless\{3,8,6\}$

Definition 2 $A=B$ if there is a one-to-one correspondence between A and B. $A<B$ if there is no one-to-one correspondence between A and B, but there is a one-to-one correspondence between A and a proper subset of B.

Thus,

$\{1,2,3\}=\{3,8,6\}$
$\{1,2\}<\{3,8,6\}$

The two sets of definitions are related in interesting ways. If $A=B$ under Definition 1, then $A=B$ under Definition 2, but the converse is not true. Similarly, if A and B are finite sets and if $A<B$ under Definition 1, then $A<B$ under Definition 2, but the converse is not true. Moreover, both $A=B$ and $A<B$ cannot be true in Definitions 1 or 2, but $A=B$ could be true on Definition 2 and $A<B$ could simultaneously be true on Definition 1.

In light of these two sets of definitions, the objector claims that the illustration of Hilbert's Hotel is ambiguous and confused. Moreover, it could be argued that the thought-experiment requires Definition 2 to be effective, but it only employs Definition 1, so it fails to support (A1).[21] To see this, let A be the original set of hotel guests prior to admission of the new guest, and let B be the set of hotel guests including the new guest. Now is $A=B$ or is $A<B$? It all depends. Given Definition 1, $A<B$ because there is an element in B (the new guest) not in A, but on Definition 2, $A=B$ because they have the same cardinality. Thus, it is ambiguous as to what is the correct way to view Hilbert's Hotel. Craig, the objector continues, is using Definition 1, but the mathematician who does not think there is any incoherence in the hotel scenario is using Definition 2.

What should one make of this objection? On the surface, it does seem that Craig's use of Hilbert's Hotel employs Definition 1. This can be seen by noting that Craig puzzles over the fact that there is a real guest who has just been given keys to a room, and his admission adds a new person to the hotel, which cannot be if there is already an infinite number of hotel guests:

> [T]here are now no more persons in the hotel than there were before: the number is just infinite. But how can this be? The proprietor just added the new guest's name to the register and gave him his keys – how can there not be one more person in the hotel than before?[22]

This would, indeed, be puzzling on Definition 1, since it stipulates that A and B are equal just in case they have the very same members, and it is clear that A has a new member not in B.

Assuming that Craig is using Definition 1, is that really a problem for his employment of Hilbert's Hotel in support of (A1), given that the objector is correct in claiming that mathematicians who do not find the scenario problematic are using Definition 2? I don't think this admission would count against Craig's argument for the following reason. He uses (A1) (and Hilbert's Hotel to support it) to argue for (A3), viz. that a beginningless temporal series of events cannot exist. Assuming Definition 1, set B is larger than set A by one member. Thus, after the new guest is added, in keeping with (A1′), this is absurd because, prior to the new guest's admission, *all* the rooms were full, and there was no spatial region available for adding new rooms or into which guests could be shifted. On the one hand, there is no reason why a new guest could not walk up to the proprietor, receive keys, and check into the hotel. But given the infinite nature of Hilbert's Hotel, this is precisely what cannot happen, and the way to resolve the absurdity, argues the Kalam defender, is to reject the possibility of Hilbert's Hotel.

By analogy with Hilbert's Hotel, Craig could argue that if God were to actualize a possible world with additional moments of time that would obtain prior to the series of events in the actual world leading up to some arbitrary time t_0, where t_0 is preceeded by an actual infinite number of earlier events, this would mean that no new moments would have been added in this alternative possible world. But that is absurd because God could surely conceive of and instantiate these additional moments of time if He so desired.

To clarify the argument in slightly different terms, suppose we take the present moment to be t_0 and grant that it has been preceded by an actually infinite set of events, each of which is set off by of some arbitrary extension, say a year. Now, suppose that God desired to instantiate an alternative possible world with the same actually infinite past leading up to t_0 in the actual world, but with five additional years added prior to that infinite series of events. By Definition 1 we have a new set of events that is larger than the one in the actual world. But how could this be? How could God instantiate such a world? By analogy with Hilbert's Hotel, there is no temporal room for any more events to have been added to the past, given that the past is temporally full and infinitely extended.[23] Moreover, if prior to the addition of five more years all the events were assigned a numeral denoting a natural number, then there would be no numerals available for the new years. Thus, it would seem that God could not instantiate the alternative possible world by creating the five additional years and placing them prior to the infinite series of events leading up to t_0 in the actual world. But this is absurd. God can conceive of and create any temporal moments He pleases. By analogy with Hilbert's Hotel, the way out of this absurdity is to deny the possibility of an infinite temporal series of events. So, even if Craig implicitly uses the first set of definitions and mathematicians use the second set, his argument could still go through.

The fact that Craig's argument may be employing Definition 1 and not Definition 2 serves to rebut an argument against Craig raised by Wes Morriston (stated in terms of adding/subtracting books from an infinite library similar to adding/subtracting guests from Hilbert's Hotel):

> Euclid's maxim [that the number of elements in a set is greater than the number in a proper subset] need not be interpreted to mean that the *number* of elements in the whole is greater than the *number* of elements in the part. There is, for example, an obvious sense in which Craig's imaginary library is 'greater' than any of its parts, and this is so despite the fact that it does not have a greater *number* of books than they. For instance, the library as a whole is 'greater' ('larger') than the part of the library containing only books numbered 3 and higher simply in virtue of the fact that it contains books numbered 0, 1, and 2 as well as all the higher numbered books. This is all by itself a perfectly legitimate sense of the word 'greater' – one that is logically independent of the question 'What is the *number* of books in the two sets?'[24]

In terms of our discussion, Morriston is correctly pointing out both that Definitions 1 and 2 are legitimate yet different ways to spell out the set-theoretic notion of '=' and '<' and that A = B could be true on Definition 2, and A < B could simultaneously be true on Definition 1. If Craig's argument employs Definition 1 and amounts to the claim that, in adding events to the past, we create a set of events B that is greater than the prior set of events A, which is impossible because there is no temporal space to add those events, then Morriston's point that A and B could be equal on Definition 2 is moot.

But what if Craig is actually employing the second set of definitions? And what of the objection that the success of the argument turns on the use of the second set, but that given those definitions, the argument actually fails? Notice that Definition 1 focuses attention on the actual identity of the members of the sets A and B, while Definition 2 abstracts from the identity of those members and focuses on the cardinality of A and B. It could be argued that what Craig really needs to make his argument work should not be tied to the specific identity of the members of the sets employed in his argument but on their cardinality, precisely because it is the sheer existence of a set with that cardinality against which he inveighs.

In fact, some things Craig says seem to indicate that he is employing the second set of definitions. After all, his emphasis seems to be on the simple fact that the number of members of A is not larger than the number of members of B because the cardinality is the same in both cases. Earlier, I offered the following sufficient condition for a set S being an actual infinite set:

(A12) A set S is actually infinite if it can be put into one-to-one correspondence with a proper subset of itself.

I also noted that the point of (A12) is not in the identity of members included in the proper subset; rather, it is in the cardinality of that subset. To support this claim, I offered an adjustment of (A12):

(A12′) A set S is actually infinite if it can be put into one-to-one correspondence with a set T that can be put into one-to-one correspondence with a proper subset of S.

With slight modifications, a thought-experiment involving Hilbert's Hotel can be formulated in keeping with (A12′) that would equally capture Craig's use of the thought-experiment. Suppose that each member of set B (the set of guests including the new one) and set A (the original set of guests) are married to one, and only one, person. If we form a set C whose members are the spouses of B, one can still puzzle over the question of how it could be that C does not contain more members than A, since C seems to have a greater number of members than A does.

Does this interpretation of Craig's argument render it a plausible thought-experiment in support of (A1)? I think one's answer to this question will depend, at least in part, on our discussion about Platonism. For now the debate focuses on Euclid's maxim that a whole is greater than any of its proper parts. Craig's argument could still be successful if it is limited to sets whose members are finite, contingent entities that can be added to or subtracted from sets, and whose members are extended in the relevant way. But it may be unsuccessful when applied to abstract objects, or so say some Platonists.

In any case, on the assumption that Definition 2 is employed by Craig, Morriston may, in fact, err when he queries Craig in this way: 'What is there to say in support of the way Craig applies Euclid's maxim to 'real-world' sets? Why should we suppose that [it] applies to *all* legitimate sets?'[25] If what I have argued is correct, then it is open to Craig, or at least to one of his defenders, to say that his arguments do not require adopting an attitude according to which the application of Euclid's maxim generates unacceptable puzzles regarding all sets whatever. Instead, one could hold, precisely because of the troublesome features of Hilbert's Hotel and related thought-experiments, that it is only when Euclid's maxim is applied to sets with members which are finite, spatially or temporally located/extended, and moveable, that problems arise. Moreover, this qualification may, in fact, remove some of the force behind intuitions in favour of denying that Euclid's maxim applies to the sets relevant to Craig's defence of (A1). It may be that those intuitions gain their plausibility when applied to sets whose members are abstract objects but lose their force for sets relevant to Craig's thought-experiments.

Finally, we could understand 'real-world sets' to mean those whose members are finite, changeable, and relevantly extended, and not those whose members are abstract objects. When Morriston asks what, exactly, Craig means by 'the real world' here, he quotes Craig as saying 'When I say that an actual infinite cannot exist, I mean "exist in the real world" or "exist outside the mind" … [w]hat I am arguing is that an actual infinite cannot exist in the real world of stars and planets and rocks and men.'[26] It may well be that Craig has other reasons for preferring conceptualism over a Platonist construal of abstract objects. But for the limited purposes of defending (A1), this preference may not be needed, at least in light of the issues addressed here.

To be sure, when Craig unpacks 'exist in the real world' as 'exist outside the mind', then this most naturally favours a conceptualist interpretation, which is likely to be Craig's actual view. But when he unpacks 'exist in the real world' as 'exist in the real world of stars and planets and rocks and men', it leaves room for one to interpret, or at least appropriate, Craig's arguments on behalf of (A1), such that they involve the claim that an actual infinite cannot exist in the spatio-temporal cosmos with finite, moveable, relevantly extended members, not that an actual infinite cannot exist in the realm of abstract objects.

In sum, the Kalam cosmological argument has been a centrepiece in the revival of interest in theistic arguments, and while it is open to a defender of the argument to employ Argument B in support of the Kalam's first premise, those who accept Argument A must provide a response to the two objections considered above. I have tried to do just that. Whether I have been successful, of course, is another matter.

Notes

1. This revival had not gone unnoticed by atheist philosophers. See Quentin Smith 'The metaphilosophy of naturalism', *Philo*, **4** (2001), 195–215.
2. Cf. William Lane Craig *Reasonable Faith* (Wheaton, IL: Crossway Books, 1994); William Lane Craig and Quentin Smith *Theism, Atheism, and Big Bang Cosmology* (Oxford: Clarendon Press, 1993).
3. Wes Morriston 'Craig on the actual infinite', *Religious Studies*, **38** (2002), 147. It is beyond the scope of my present concerns to consider all of Morriston's critique and, thus, I shall only examine Morriston's arguments that are relevant to the scope of issues addressed here.
4. See J. P. Moreland 'Agent causation and the Craig/Grünbaum debate about theistic explanation of the initial singularity', *American Catholic Philosophical Quarterly*, **71** (1997), 539–554.
5. Craig *Reasonable Faith*, 95–96.
6. Nominalists and extreme nominalists typically employ an epistemic notion of 'abstract object', according to which an object is abstract just in case it is brought before the mind by selective inattention to other items surrounding the object and by focusing attention on the object itself.

7. It may be possible to reconcile a commitment to abstract objects as necessary beings with the traditional theistic view that God is the only necessary being by drawing a distinction between being supremely necessary and being strongly necessary. Something is *supremely necessary* just in case it exists in all possible worlds and it does not depend on anything else for its existence. Something is *strongly necessary* just in case it exists in all possible worlds and depends on something else for its existence that has the property of necessarily sustaining it.
8. See D. M. Armstrong *Universals and Scientific Realism*, vol. 1: *Nominalism and Realism* (Cambridge: Cambridge University Press, 1978); vol. 2: *A Theory of Universals* (Cambridge: Cambridge University Press, 1978); *idem* 'Naturalism, materialism, and first philosophy', *Philosophia*, **8** (1978), 261–276; *idem* 'Can a naturalist believe in universals?' in Edna Ullmann-Margalit (ed.) *Science in Reflection* (Boston, MA: Kluwer Academic Publishers, 1988), 103–115; *idem Universals: An Opinionated Introduction* (Boulder, CO: Westview Press, 1989); *idem A World of States of Affairs* (Cambridge: Cambridge University Press, 1997).
9. See J. P. Moreland *Universals* (Audley, UK: Acumen Press; Montreal and Kingston, ONT: McGill-Queen's University Press, 2001), ch. 4.
10. See Chris Swoyer 'Theories of properties: from plenitude to paucity', in James E. Tomberlin (ed.) *Philosophical Perspectives 10: Metaphysics, 1996* (Cambridge, MA: Blackwell, 1996), 243–264, 245.
11. *Ibid*., 260.
12. Morriston 'Craig on the actual infinite', 157; cf. 164.
13. Craig *Reasonable Faith*, 97.
14. Regarding this second feature, it may be that the problem does not generalize to sets whose finite, contingent members are not extended is some relevant way. Thus, if there are unextended monads of space or time, it may be that there could be an infinite number of them even if they are finite, contingent entities. Of course, there are serious difficulties with embracing such monads (e.g. the problem of generating spatially or temporally extended entities, including extended regions of space and time, from unextended units), but for the purposes of defending (A1) in light of the Platonist objection, I leave the matter of infinite collections of such monads as an open question.
15. If could be argued that there are not really two separate intuitions on the grounds that (ii) is actually the ground for (i). Now, in fact, it may be that some would appeal to (ii) as the ground for (i). But it is consistent for someone to accept (i) and have no view of (ii). There is a parallel to this dialectic in debates about the use of the modal argument (disembodied existence or body switches are strongly conceivable and this provides defeasible justification for belief in a substantial non-bodily ego). Stewart Goetz has argued that the modal argument is actually grounded in the epistemically basic fact that we are aware of ourselves as being simple and our bodies as being complex. See his 'Modal dualism: a critique', in Kevin Corcoran (ed.) *Soul, Body and Survival* (Ithaca, NY: Cornell University Press, 2001), 89–104. Goetz's position is analogous to one who would claim that (i) is grounded in and not autonomous from (ii). Charles Taliaferro has responded by claiming that the force of the modal argument rests on the intrinsic plausibility of detailed narratives involved in the argument. See Charles Taliaferro 'Sensibility and possibility: a defense of thought experiments', *Philosophia Christi*, series 2, **3** (2001), 403–420. Taliaferro's position is analogous to one who could claim that (i) is autonomous from (ii). Now, even if it turns out that Goetz is correct regard the modal argument, it does not follow that all relevantly similar dialectic forms must follow this pattern. For it may well be that certain features of a thought experiment are the problematic aspects besides some alleged intuitive principle widely thought to be epistemically basic to the thought experiment's troublesome nature.
16. Morriston 'Craig on the actual infinite', 148–149.
17. *Ibid*., 148.
18. *Ibid*., 149.
19. J. P. Moreland 'Theories of individuation: a reconsideration of bare particulars', *Pacific Philosophical Quarterly*, **79** (1998), 251–263; *idem* 'Issues and options in individuation', *Grazer Philosophische Studien*, **60** (2000), 31–54.
20. The objection has been raised by John LeFever in an unpublished paper entitied 'Infinite sets'. Since he applies his argument against a thought-experiment involving a library with an infinite collection of books, I shall modify LeFever's discussion slightly to apply it to Hilbert's Hotel.
21. LeFever does not make this additional point in his paper.
22. Craig *Reasonable Faith*, 95.

23. It is irrelevant to this argument, that given a finite past, more and more events could be added as time goes on to form a growing history. The argument under consideration conceives of a possible world with an infinite past in which additional events have been added prior the actual series of events – alleged to be infinite in number – leading up to an arbitrary event in the actual world.
24. Morriston 'Craig on the actual infinite', 154–155.
25. *Ibid.*, 154.
26. Cited in *ibid.*, 156.